

Flow and Heat Transfer of a MHD Viscoelastic Fluid in a Channel with Stretching Walls: Some Applications to Haemodynamics

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Abstract

Of concern in the paper is a study of steady incompressible viscoelastic and electrically conducting fluid flow and heat transfer in a parallel plate channel with stretching walls in the presence of a magnetic field applied externally. The flow is considered to be governed by Walter's liquid B fluid. The problem is solved by developing a suitable numerical method. The results are found to be in good agreement with those of earlier investigations reported in existing scientific literatures. The study reveals that a back flow occurs near the central line of the channel due to the stretching walls and further that this flow reversal can be stopped by applying a strong external magnetic field. The study also shows that with the increase in the strength of the magnetic field, the fluid velocity decreases but the temperature increases. Thus the study bears potential applications in the study of the haemodynamic flow of blood in the cardiovascular system when subjected to an external magnetic field.

Keywords: Non-Newtonian fluid, MHD flow, Viscoelasticity, stretching walls, Heat transfer

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1 Introduction

In recent years, the study of magnetohydrodynamic (MHD) flow of blood through artery has gained considerable interest because of its physiological applications. Investigations on MHD flow and heat transfer of non-Newtonian fluids over a stretching sheet also find many important applications in engineering and industry. For example in the extrusion of a polymer sheet from a die, the sheet is sometimes stretched. The properties of the end product depend considerably on the rate of cooling. By drawing such a sheet in a visco-elastic electrically conducting fluid subjected to the action of a magnetic field, the rate of cooling can be controlled and the final product can be obtained with desired characteristics. Crane[1] investigated the problem of steady two-dimensional incompressible boundary layer flow engendered by the stretching of an elastic flat sheet which moves in its plane with a velocity varying linearly with distance from a fixed point due to application of a uniform stress. Misra et al.[2] studied the Hall effect on the steady MHD boundary layer flow of an incompressible viscous and electrically conducting fluid past a stretching surface in the presence of a uniform transverse magnetic field.

Nomenclature

η	non-dimensional distance
σ	electrical conductivity
ρ	density
γ	kinematic viscosity
B_0	applied magnetic field
k_0	coefficient of visco-elasticity
θ	dimensionless temperature
a	channel half width
(u,v)	velocity components along x and y directions, respectively
b	a constant of proportionality
T	temperature variable
P_r	Prandtl number
M	Magnetic number
K_1	visco-elastic parameter
K	thermal diffusivity

A consistent mathematical model for the unsteady flow of blood through arteries was put forward by Misra and Chakravarty [3] in which the blood was treated as a Newtonian viscous incompressible fluid by paying due attention to the orthotropic material behaviour of the wall tissues. Misra et al. [4] also conducted another theoretical study concerning blood flow through a stenosed segment of an artery, where they modelled the artery as a non-linearly viscoelastic tube filled with a non-Newtonian fluid representing blood.

The effects of a uniform transverse magnetic field on the motion of an electrically conducting fluid past a stretching surface were investigated by Pavlov[5], Kumari et al.[6], Anderson[7], Datti et al.[8] and Chamkha[9]. Rajagopal et al.[10, 11] carried out studies on the boundary layer and non-similar boundary layer flow of an incompressible homogeneous non-Newtonian fluids of second order over a stretching sheet in the presence of a uniform free stream. These researchers restrict their analyses to hydromagnetic flow of blood through an artery with stretching walls of the vessel and heat transfer. Anderson et al.[12] studied the boundary layer flow of an electrically conducting incompressible fluid obeying the Ostwald-de-Waele power-law model in the presence of a transverse magnetic field due to the stretching of a plane sheet.

Misra et al.[13, 14] investigated the steady flow of an incompressible viscoelastic and electrically conducting fluid in a parallel plate channel in the presence of a uniform transverse magnetic field. As illustrations of the applicability of these analyses they studied the flow of blood in arteries with stretchable walls, by considering blood as a non-Newtonian fluid.

It was observed by Fukada and Kaibara[15], Thurston[16] and Stoltz et al.[17] that under certain conditions blood exhibits visco-elastic behaviour which may be attributed to the visco-elastic properties of the individual red cells and the internal structures formed by cellular interactions. Since blood is electrically conducting, its flow in the cardiovascular system is likely to be influenced by a magnetic field.

Modelling of hyperthermia-induced temperature distribution requires an accurate description of the mechanism of heat transfer. It is reported in [18] that blood flow affects the thermal response of living tissues. The heat exchange between living tissues and blood network that passes through it depends on the geometry of the blood vessels and the flow variation of blood.

Craciunescu and Clegg [19] studied the effect of oscillatory flow upon the resulting temperature distribution of blood and convective heat transfer in rigid vessels. The importance of different types of blood vessels in the process of bioheat transfer has been intensively studied by Weinbaun et al.[20] and Jiji et al [21]. Cavaliere [22] and his co-workers examined application of heat to human tumors in the extremities by local perfusion with warm blood. They made an observation that heat alone can lead to total regression of melonomas and sarcomas and an increase in survival patients. Shitzer and Eberhart [23] presented various theoretical frameworks that can be used to estimate heat transfer from an external or internal source to a tissue. They predicted resulting temperature distributions in normal tissues of various mammals during hyperthermia. This information is important for improving tumor detection by designing heating protocols for hyperthermic treatment.

In the present study, we investigated the problem of steady MHD flow of a visco-elastic fluid in a parallel plate channel permeated by a uniform transverse magnetic field in a situation where the surface velocity of the channel varies linearly with distance from the origin. The motivation of this study is to analyze the flow of blood in arteries whose walls are stretchable and the flow field is governed by the non-Newtonian behaviour depicted by Walter's liquid B fluid model. The equations of motion of non-Newtonian fluids considered by us are highly non-linear. These equations are one order higher than the order of the Navier-Stokes equations. Due to the complexity of these equations, finding exact solution is rather difficult. We have, therefore, developed a numerical method to solve the problem. In addition, in view of the information stated in the preceding paragraph, we have also performed a heat transfer analysis of the problem in question.

2 Analysis

Let us consider the steady laminar flow of an incompressible and electrically conducting visco-elastic fluid in a parallel plate channel bounded by the planes $y = \pm a$. The flow is driven by the stretching of the channel walls such that the surface velocity of each wall is proportional to the distance from the origin (cf. Fig. 1).

A uniform magnetic field of strength B_0 is imposed along the normal to the channel walls i.e., parallel to y-axis, the electrical conductivity σ being assumed constant. The flow is considered to be governed by the rheological equation of state derived by Beard and Walter [24]. The

steady two-dimensional boundary layer equations for this flow in usual notation are

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \gamma \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u - k_0 \left(u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right) \quad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

where (u, v) are the fluid velocity components along x-and y- directions respectively. $\rho, \gamma, B_0, \sigma$, and k_0 are respectively the density, kinematic viscosity, applied magnetic field, electrical conductivity and coefficient of visco-elasticity. The induced magnetic field produced by the motion of the fluid in the presence of the external magnetic field B_0 is assumed negligible. Considering the flow to be symmetric about the center line $y=0$ of the channel, we focus our attention to the flow in the region $0 \leq y \leq a$ only.

The boundary conditions applicable to the flow problem are

$$u = bx, \quad v = 0 \quad \text{at} \quad y = a \quad (3)$$

$$\frac{\partial u}{\partial y} = 0, \quad v = 0 \quad \text{at} \quad y = 0 \quad (4)$$

with $b > 0$.

Equations (1) and (2) admit of a self similar solution of the form

$$u = bx f'(\eta), \quad v = -ab f(\eta) \quad \text{and} \quad \eta = \frac{y}{a}. \quad (5)$$

Now we introduce the following non-dimensional variables

$$x^* = \frac{x}{a}, \quad \eta = \frac{y}{a}, \quad u^* = \frac{u}{ab}, \quad v^* = \frac{v}{ab}. \quad (6)$$

In terms of these dimensionless variables, equation (5) can be put as

$$u^* = x^* f'(\eta), \quad v^* = -f(\eta). \quad (7)$$

Clearly u and v satisfy the continuity equation (2) identically. Substituting these new variables in equation (1), we have

$$f'^2 - f f'' = f''' - M f' - K_1 \{ 2 f' f''' - f f^{iv} - f''^2 \} \quad (8)$$

where $K_1 = \frac{k_0 b}{\gamma}$ is the viscoelastic parameter and $M = \frac{\sigma B_0^2}{\rho b}$ is the magnetic parameter, $\gamma = ba^2$.

With the use of the transformation (7), the boundary conditions (3) and (4) read

$$f'(\eta) = 1, \quad f(\eta) = 0 \quad \text{at} \quad \eta = 1 \quad (9)$$

$$f''(\eta) = f(\eta) = 0 \quad \text{at} \quad \eta = 0 \quad (10)$$

3 Perturbation Analysis

Since K_1 is assumed to be small, to solve the equation (8) we follow a perturbation expansion approach by writing

$$f = f_0(\eta) + K_1 f_1(\eta) + K_1^2 f_2(\eta) + \dots \quad (11)$$

Substituting this into the equation (8) and equating like powers of K_1 , ignoring quadratic and higher powers of K_1 , we obtain

$$f_0'' - M f_0' = f_0'^2 - f_0 f_0'' \quad (12)$$

$$\text{and } f_1''' + f_0 f_1'' - 2 f_0' f_1' + f_1 f_0'' - M f_1' = 2 f_0' f_0''' - f_0 f_0^{iv} - f_0''^2. \quad (13)$$

Using (11) in (9) and (10), the boundary conditions for f_0 and f_1 become

$$f_0(0) = f_0''(0) = 0, \quad f_0'(1) = 1, \quad f_0(1) = 0 \quad (14)$$

$$f_1''(0) = f_1(0) = f_1'(1) = f_1(1) = 0 \quad (15)$$

The foregoing system of equations arising out of the problem of MHD visco-elastic fluid flow past a stretching surface under consideration does not admit of an exact analytical solution. We have, therefore, developed a numerical method suitable for solving the said equations.

4 Heat Transfer Analysis

The heat transfer equation for the boundary layer approximation can be written as

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = K \frac{\partial^2 T}{\partial y^2}, \quad (16)$$

(by neglecting viscous and ohmic dissipation) where $T(x, y)$ is the temperature at any point and K is the thermal diffusivity of the fluid. The boundary conditions for heat transfer are taken to be

$$T = T_w \quad \text{at } y = a \quad (17)$$

$$\frac{\partial T}{\partial y} = 0 \quad \text{at } y = 0 \quad (18)$$

where T_w is a constant.

We introducing the non-dimensional temperature variable

$$\theta = \frac{T}{T_w}. \quad (19)$$

Using (6), (7) and (19), the equations (16), (17) and (18) can be rewritten in the form

$$\theta'' + P_r f \theta = 0 \quad (20)$$

$$\theta'(\eta) = 0 \quad \text{at } \eta = 0 \quad (21)$$

$$\theta(\eta) = 1 \quad \text{at } \eta = 1 \quad (22)$$

where $P_r = \frac{\gamma}{K}$ is Prandtl number.

It is worthwhile to mention here that as in the case of velocity distribution, the equation (16) that governs the temperature distribution also admits of a similarity solution.

After $f(\eta)$ is determined by solving the solutions (12) and (13), the equation (20) is solved numerically subject to the boundary conditions (21) and (22) using the finite difference technique. The temperature distribution $\theta(\eta)$ is thus characterised by three non-dimensional parameters M, K_1 and P_r .

5 Numerical Method

One of the most commonly used numerical methods is the finite difference technique, which has better stability characteristics, and is relatively simple, accurate and efficient. Another essential feature of this technique is that it is based on an iterative procedure and a tridiagonal matrix manipulation. This method provides satisfactory results, but it may fail when applied to problems in which the differential equations are very sensitive to the choice of initial conditions. For solving the equations (12) and (13) subject to the boundary conditions (14) and (15), we developed a finite difference technique as briefly described below.

Substituting $F = f'_0$ in (12) and (14), we get

$$F'' + f_0 F' - F^2 - MF = 0 \quad (23)$$

$$F'(0) = 0, \quad F(1) = 1, \quad f_0(0) = 0, \quad f_0(1) = 0 \quad (24)$$

Similarly, writing $G = f'_1$ in (13) and (15) yields

$$G'' + f_0 G' - 2f'_0 G + f_1 f''_0 - MG = 2f'_0 f'''_0 - f_0 f''_0{}^2 - f''_0{}^2 \quad (25)$$

and

$$G'(0) = G(1) = 0, \quad f_1(0) = f_1(1) = 0. \quad (26)$$

Using central difference scheme for derivatives with respect to η , we can write

$$(V')_i = \frac{V_{i+1} - V_{i-1}}{2\delta\eta} + O((\delta\eta)^2) \quad (27)$$

$$\text{and} \quad (V'')_i = \frac{V_{i+1} - 2V_i + V_{i-1}}{(\delta\eta)^2} + O((\delta\eta)^2) \quad (28)$$

where V stands for F , G or T , i is the grid-index in η -direction with $\eta_i = i * \delta\eta$; $i = 0, 1, 2, \dots, m$ and $\delta\eta$ is the increment along the η -axis. Newton's linearization method has been applied to linearize the discretized equations as follows. When the values of the dependent variables at the n^{th} iteration are known, the corresponding values of these variables at the next iteration are obtained by using the equation

$$V_i^{n+1} = V_i^n + (\Delta V_i)^n \quad (29)$$

where $(\Delta V_i)^n$ represents the error at the n^{th} iteration, $i = 0, 1, 2, \dots, m$.

6 Results and Discussions

In order to study the MHD fluid flow and heat transfer, under the influence of an applied magnetic field, the described numerical technique has been developed and used to solve the differential equations (12), (13) and (20) subject to the boundary conditions (14), (15) and (21-22).

For numerical solution it is necessary to assign some numerical values to the parameters involved in the problem under consideration. A realistic case is considered in which the fluid is blood. We find that the magnetic parameter M is approximately 500 when the system is under the influence of a strong magnetic field of strength $B_0 = 8T$ (tesla) and the blood density $\rho = 1050kg/m^3$ and the electrical conductivity of blood, $\sigma = 0.8s/m$. As in [13], we consider $M=0$ to 600 and $K_1=0.005, 0.001, 0.01, 0.05, 0.1$ and for a human body temperature, $T = 310^0K$, the value of $Pr=21$ is considered for blood (cf. [25, 26]). For the sake of comparison, we have also examined the cases where $Pr=1, 7$. The stability of the numerical scheme has been tested by repeating the computational work by considering different mesh sizes, viz. $\delta\eta = 0.05, 0.025, 0.02$ and 0.0125 . Table-1 and 2 shows that there is no significant departure of the

results obtained by taking $\delta\eta = 0.025$ in comparison to those obtained by reducing the mesh size further. With this observation the entire numerical work presented here has been carried out by taking the mesh size $\delta\eta = 0.025$ with 41 grid points.

Table-1. Values of $f'(\eta)$ for different mesh sizes

η	$\delta\eta = 0.05$	$\delta\eta = 0.025$	$\delta\eta = 0.020$	$\delta\eta = 0.0125$
0.0	-0.10247923	-0.10015714	-0.10015714	-0.10004795
0.2	-0.10020489	-0.09987529	-0.09987529	-0.09952314
0.4	-0.09985562	-0.09772152	-0.09772152	-0.09745587
0.6	-0.08270367	-0.08179329	-0.08179329	-0.08152171
0.8	0.04269458	0.03639101	0.03639101	0.03583981

Table-2. Values of $-f(\eta)$ for different mesh sizes

η	$\delta\eta = 0.05$	$\delta\eta = 0.025$	$\delta\eta = 0.020$	$\delta\eta = 0.0125$
0.2	0.02048278	0.02001394	0.02001394	0.02098822
0.4	0.04077159	0.03984322	0.03984322	0.03959541
0.6	0.05963056	0.05831033	0.0583103	0.05875632
0.8	0.06804608	0.06681512	0.06681512	0.06631246

Figs. 2 and 3 give comparison between our results and those reported by Misra et al. [13] who considered blood as a second-grade fluid, for $M=100$, $K_1=0.005$. These figures depict that the variation of the axial and normal velocities of the fluid in the case of the present study are in good agreement with those of Misra et al. [13] qualitatively as well as quantitatively. Figs. 4-9 illustrate the variation of the dimensionless velocity components $u^* = x^* f'(\eta)$ and $v^* = -f(\eta)$ for a given cross-section of the channel (i.e., for a fixed value of x^*). It reveals from Figs. 4 and 6 that the velocity component normal to the channel wall decreases monotonically as the magnetic field strength increases, while from Figs. 5 and 7 we find that the axial velocity component is positive near the vessel wall ($\eta = 1$) and for each value of M reversal of flow takes place in the region adjacent to the central line of the channel. The values of η at which the axial velocity vanishes are given in Table-3 for different values of the magnetic parameter M .

Table - 3. Values of η where axial velocity has a vanishing value

M	0	100	200	300	400	500	600
η	0.58	0.77	0.81	0.82	0.84	0.85	0.86

It may be noted that for values of $M > 200$, the points where the reverse flow set in are quite close to each other. The interesting observation of flow reversal owes to its origin to the stretching of the channel wall. It is also revealed that in the vicinity of the boundary layer, the axial velocity decreases with the increase in the magnetic field strength. This observation leads to a very important suggestion that one can avoid flow reversal (studied here) by applying a sufficient strong magnetic field.

Figs. 8 and 9 depict the spatial variation of velocity components for different values of the viscoelastic parameter K_1 . From these two figures one can a very important observation that for a purely hydrodynamic flow (in the absence of any magnetic field) the velocity of blood decreases with the increase in the value of the viscoelastic parameter, but under the action of a sufficient strong magnetic field, velocity is least affected by blood viscoelasticity.

Figs. 10-11 give the spatial variation of temperature for different values of the magnetic parameter M and the different values of the viscoelastic parameter K_1 . From these figures we learn that the temperature increases with the increase in the magnetic field strength and the viscoelastic parameter. It thus turns out that under the action of a magnetic field, in an electrically conducting fluid (e.g. blood), there develops a resistive force (Lorentz force) which causes impedance of flow and enhancement of temperature. Further, Fig. 12 shows that under the action of a sufficient strong magnetic field the temperature of blood decreases with the increase in Prandtl number.

7 Summary and Conclusions

The flow of a magnetohydrodynamic fluid in a channel with stretching walls and an associated problem of heat transfer have been the concern of this paper. The study is particularly motivated towards the flow and heat transfer of blood in a vessel having stretching wall. It bears the potential to examine the variation of blood velocity as the magnetic field strength (M),

fluid (blood) viscoelasticity (measured by K_1) and thermal diffusivity (which is related to the Prandtl number Pr). The study reveals that the flow reversal can be eliminated and the vessel wall temperature can be controlled by applying a sufficiently strong magnetic field.

The observation of the study that temperature increases as the strength of the magnetic field increases is likely to be useful in the development of new heating methods. It can also have significant clinical application of hyperthermia in cancer therapy. The objective of hyperthermia in cancer therapy is to raise the temperature of cancerous tissues above a therapeutic value 42°C while maintaining the surrounding normal tissue at sublethal temperature [27]. The data reported here through different graphs should be of interest to medical persons in the treatment of various health problems, including arrest of cancerous tumor growths and to achieve the goal for monitoring of the tumor and normal tissue temperatures. .

Acknowledgements: *The authors wish to express their deep sense of gratitude to the reviewers of the original manuscript for their kind suggestions based upon which the present version of the paper has been prepared. One of the authors (J. C. Misra) is thankful to the Alexander von Humboldt Foundation, Germany for supporting this investigation.*

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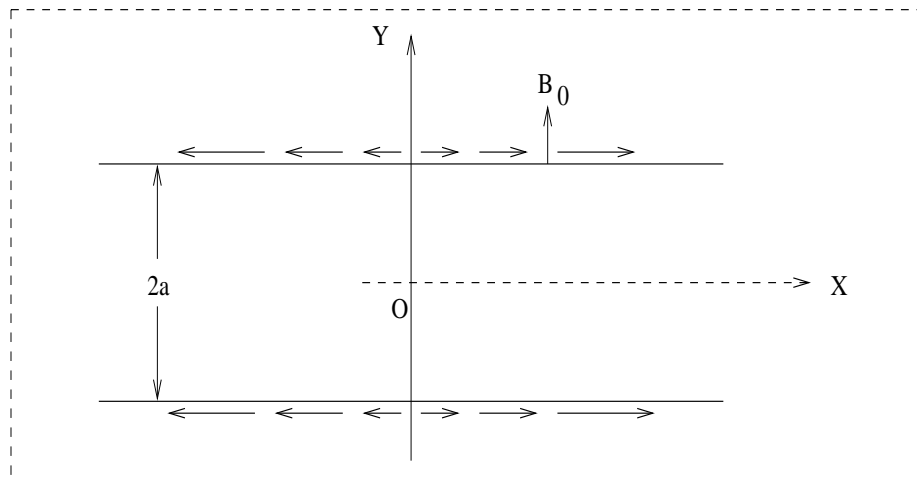


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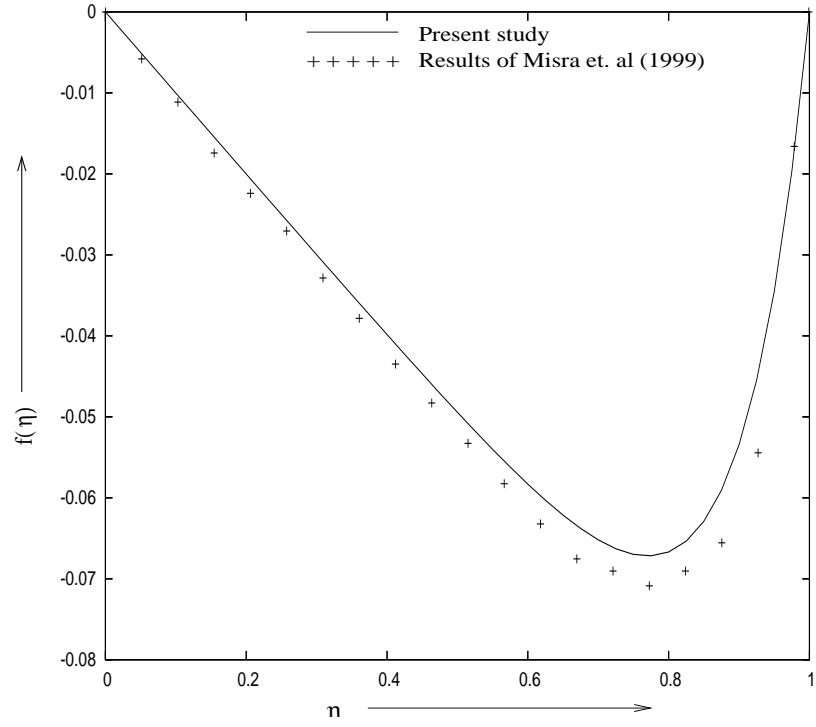


Fig. 2 Variation of $f(\eta)$ with η for $M=100$ and $K_1=0.005$

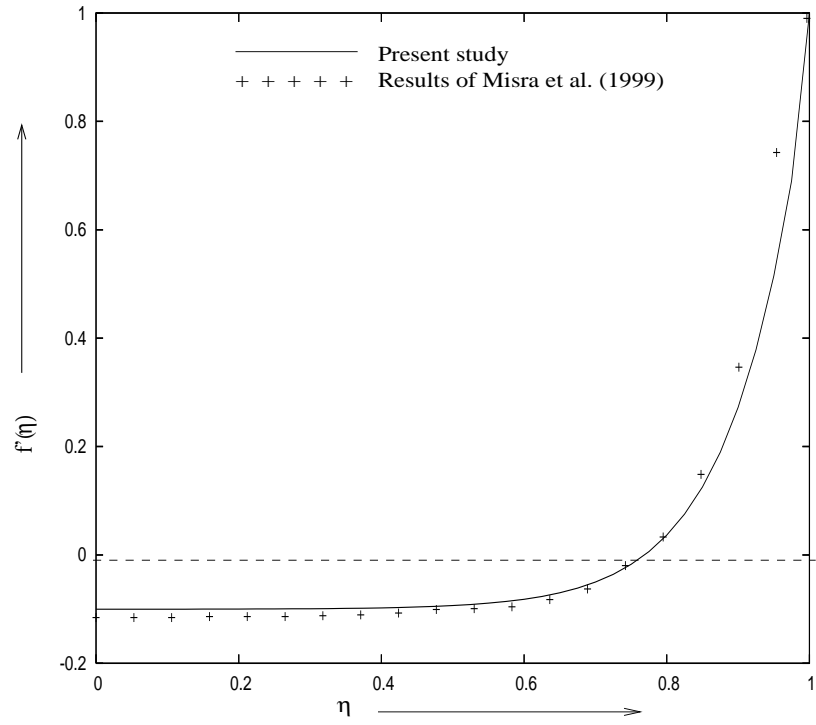


Fig. 3 Variation of $f'(\eta)$ with η for $M=100$ and $K_1=0.005$

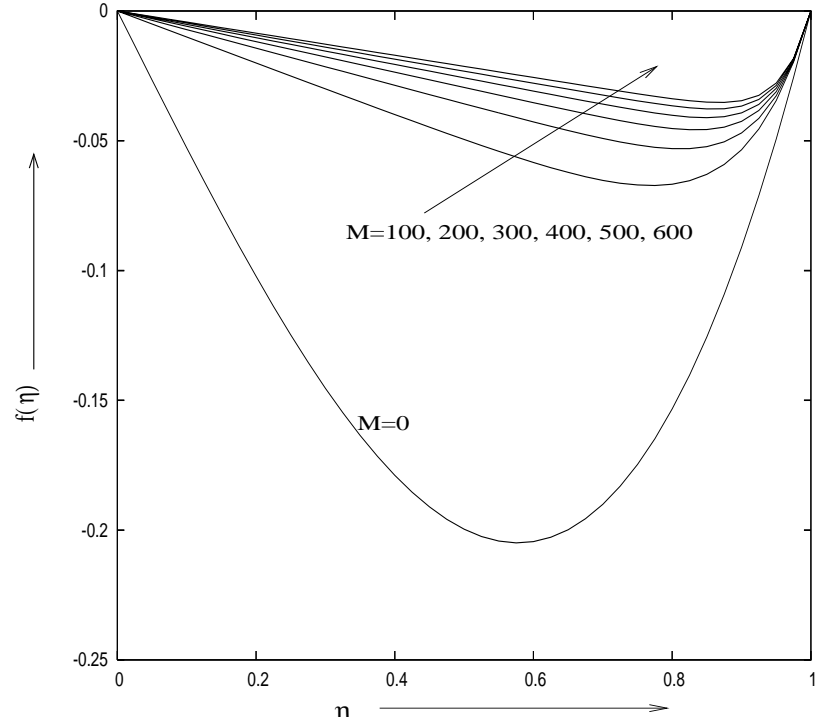


Fig. 4 Variation of $f(\eta)$ with η for different values of M and $K_1=0.001$

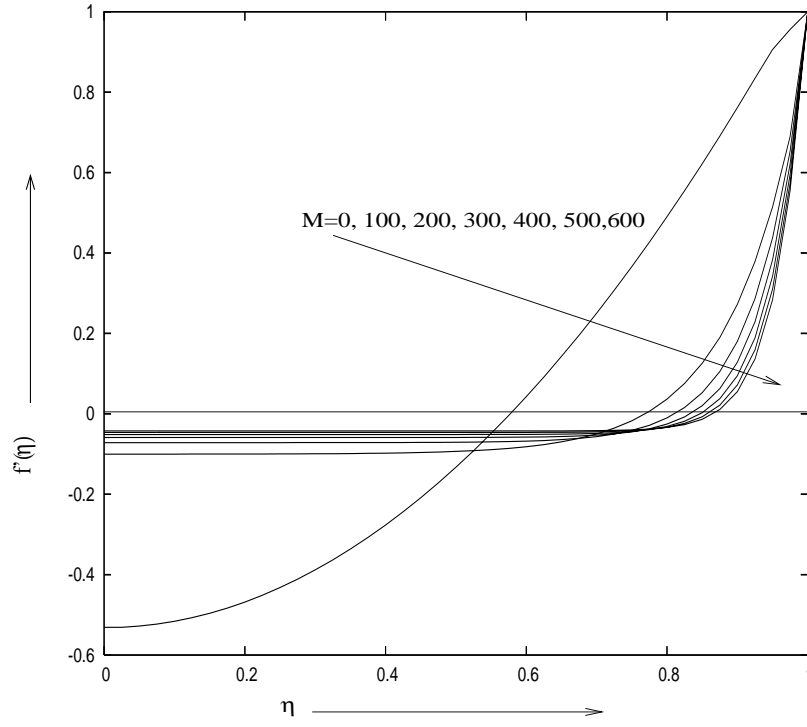


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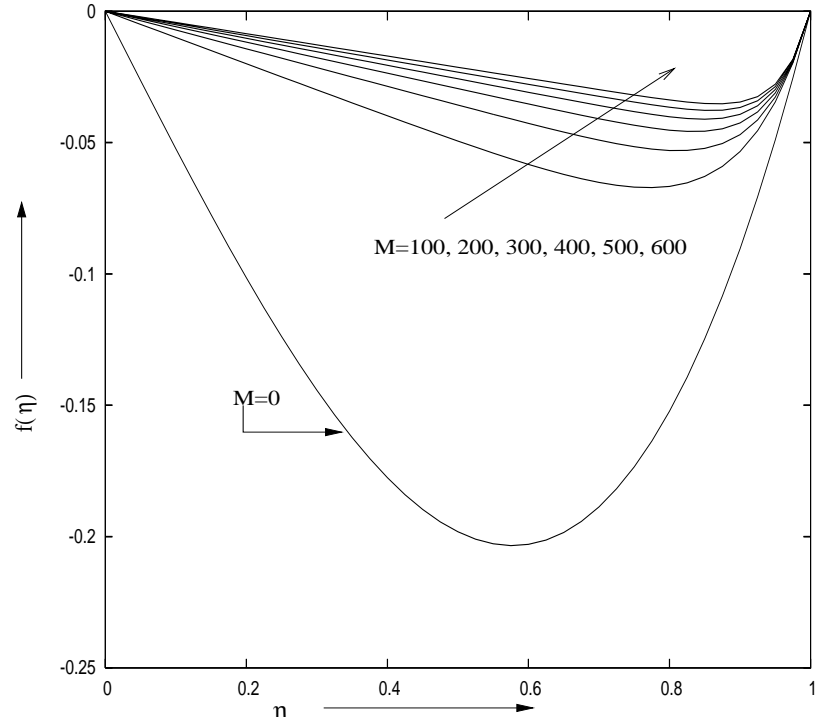


Fig. 6 Variation of $f(\eta)$ with η for different values of M and $K_1=0.005$

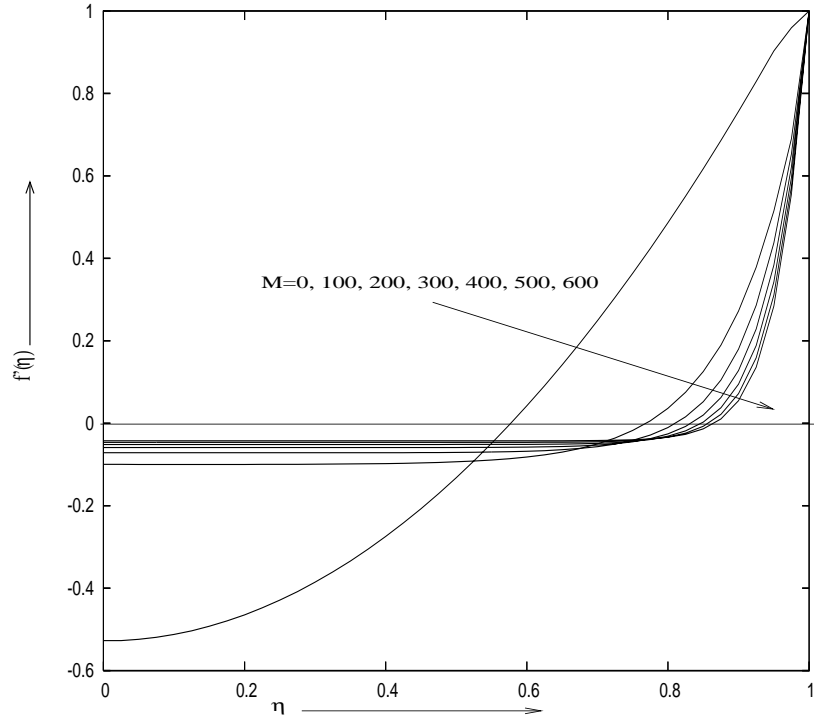


Fig. 7 Variation of $f'(\eta)$ with η for different values of M and $K_1=0.005$

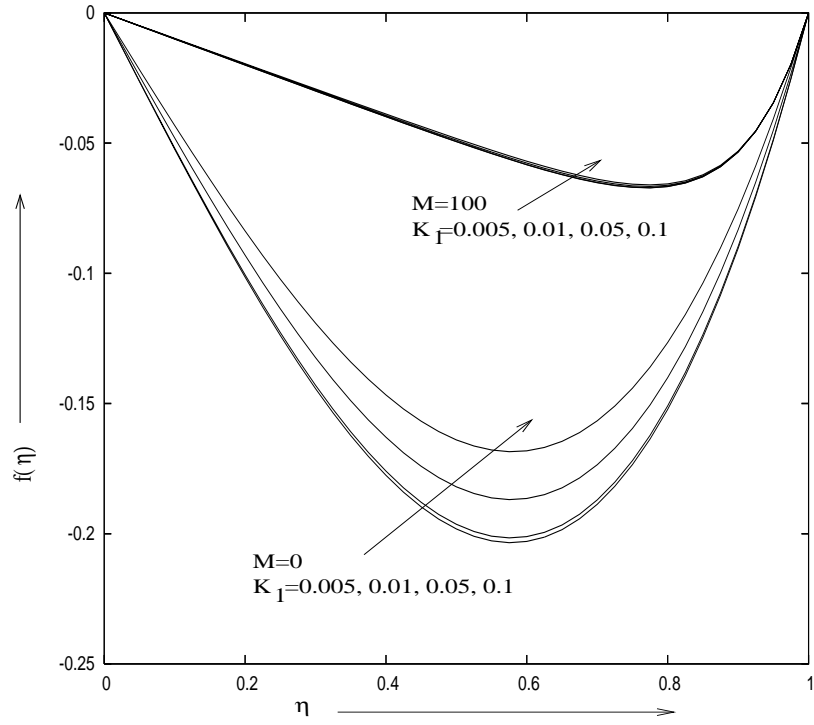


Fig. 8 Variation of $f(\eta)$ with η for different values of K_1

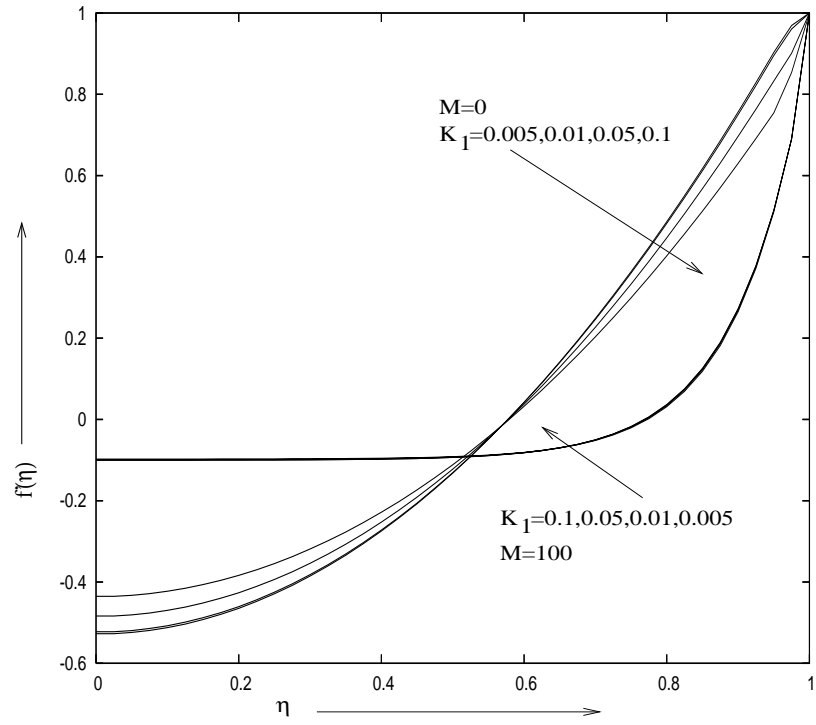


Fig. 9 Variation of $f'(\eta)$ with η for different values K_1

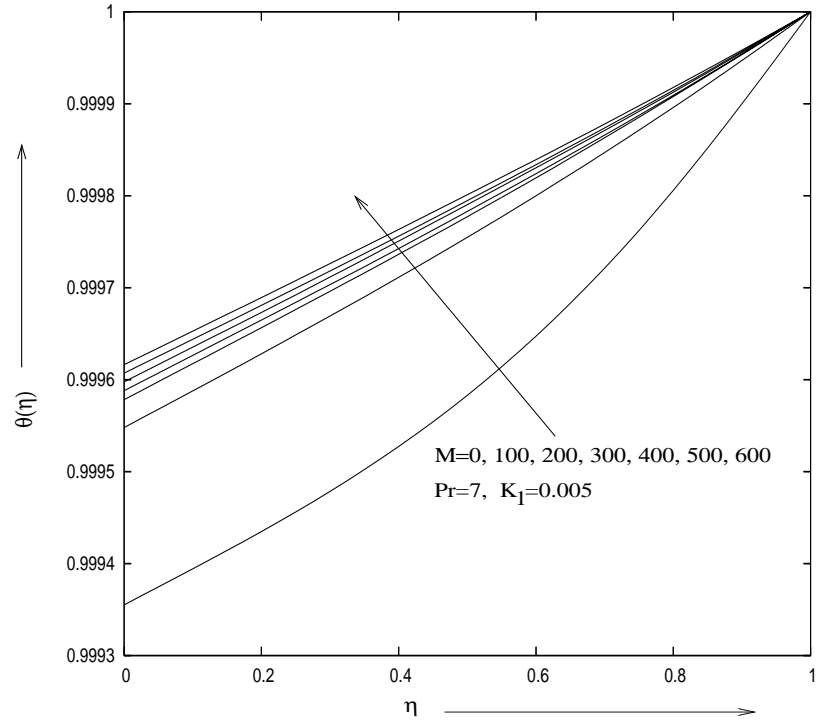


Fig. 10 Variation of non-dimensional temperature with η for different values of M

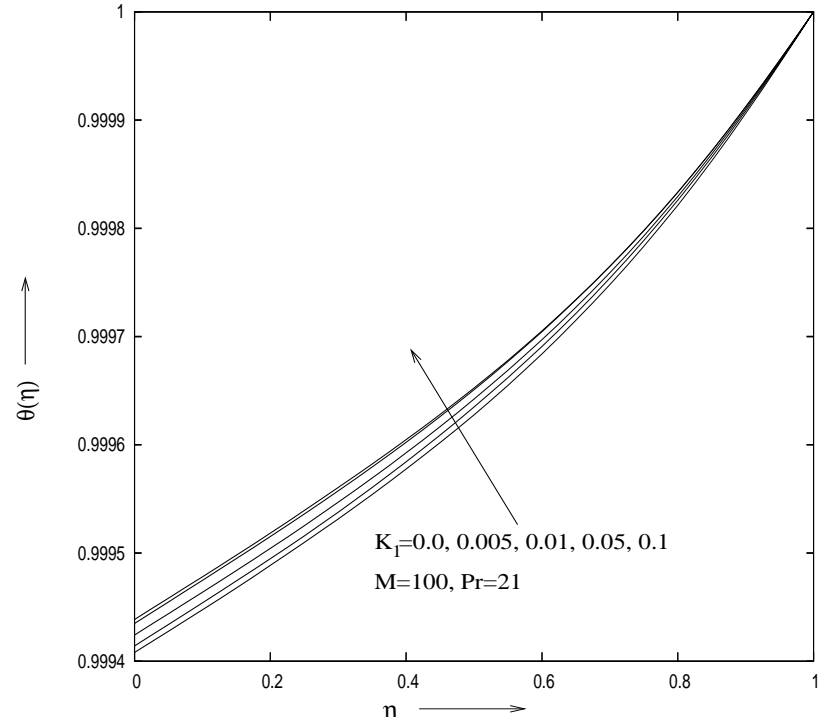


Fig. 11 Variation of non-dimensional temperature with η for different values of K_1

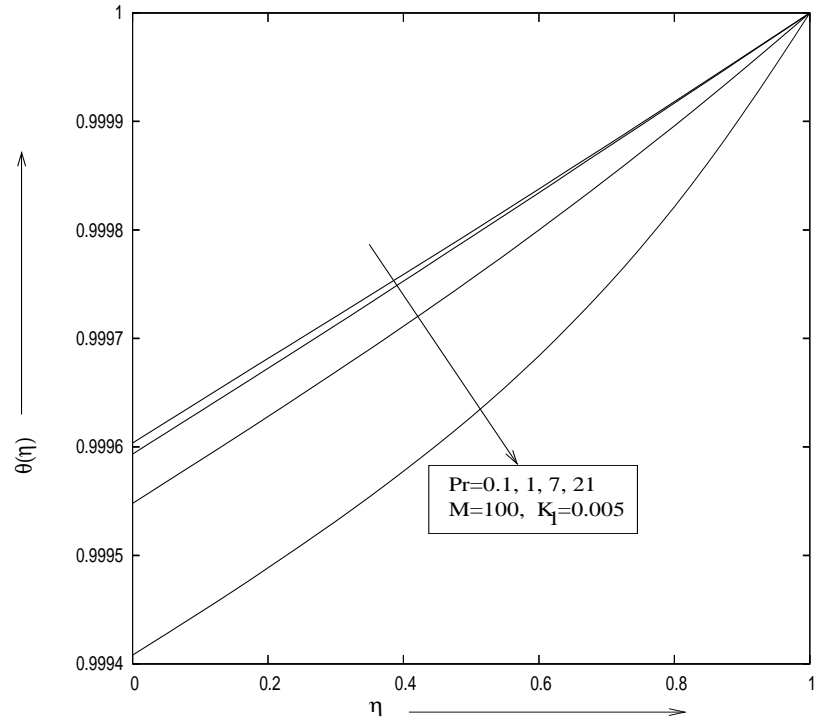


Fig. 12 Variation of non-dimensional temperature with η for different values of Pr